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كمال توفيق

Lecture #0
Appendix: A

Numbers, Inequalities, and Absolute Values

الاعداد.المتباينات .والقيم المطلقة

MATH 110

Sets المجموعات

A set is a collection of well defined objects, and these objects are called the elements of the set.

Notations

If S is a set and a is an element of S, we write the notation $a \in S$

If a is not an element of S, we write the notation $a \notin S$

Intervals الفترات

Intervals are sets of real numbers. For example if a < b, then the **open interval from a to b** consists of all numbers between a and b and is denoted by the symbol (a, b).

Using set-builder notation, we can write $(a, b) = \{x \mid a < x < b\}$

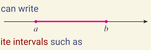
Notice that the endpoints of the interval—namely, a and b—are excluded. This is indicated by the round brackets ( ) and by the open dots in Figure shown.

Closed interval الفترات المغلقة

If the **endpoints of the interval are included**. This is indicated by the square brackets [ ] and by the solid dots in the Figure .

Using set-builder notation, we can write

$[a, b] = \{x \mid a \leqslant x \leqslant b\}$



We also need to consider infinite intervals such as

$$(a, \infty) = \{x \mid x > a\}$$

The notation (a, ∞) stands for the set of all numbers that are greater than a. ∞ is the symbol ∞ simply indicates that the interval extends indefinitely far in the positive direction.

Notation	Set description	Picture
(a, b)	$\{x \mid a < x < b\}$	a b
$[a, b]$	$\{x \mid a \leqslant x \leqslant b\}$	a b
$[a, b)$	$\{x \mid a \leqslant x < b\}$	a b
$(a, b]$	$\{x \mid a < x \leqslant b\}$	a b
(a, ∞)	$\{x \mid x > a\}$	a
$[a, \infty)$	$\{x \mid x \geqslant a\}$	a
$(-\infty, b)$	$\{x \mid x < b\}$	b
$(-\infty, b]$	$\{x \mid x \leqslant b\}$	b
$(-\infty, \infty)$	$\mathbb{R}$ (set of all real numbers)	

Inequalities المتباينات

When working with inequalities, note the following rules.

2 Rules for Inequalities

1. If $a < b$, then $a + c < b + c$.
2. If $a < b$ and $c < d$, then $a + c < b + d$.
3. If $a < b$ and $c > 0$, then $ac < bc$.
4. If $a < b$ and $c < 0$, then $ac > bc$.
5. If $0 < a < b$, then $1/a > 1/b$.

Example 1

Solve the inequality in terms of intervals and illustrate the solution set on the real number line.

1) $2x + 7 > 3$
2) $2x + 1 \leq 5x - 8$
3) $-5 \leq 3 - 2x < 9$
4) $x^2 < 2x + 8$
5) $x^2 \geq 25$

Absolute Value القيمة المطلقة

The absolute value of a number a, denoted by $|a|$ is **the distance from a to 0 on the real number line.** Distances are always positive or 0, so we have

$$|a| \geqslant 0 \qquad \text{for every number } a$$

For example,

$|3| = 3 \quad |-3| = 3 \quad |0| = 0 \quad |\sqrt{2} - 1| = \sqrt{2} - 1 \quad |3 - \pi| = \pi - 3$

In general, we have

$$|a| = a \quad \text{if } a \geqslant 0$$
$$|a| = -a \quad \text{if } a < 0$$

Example 2

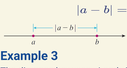
Express without using the absolute-value symbol.

$|2x - 6|$ $\qquad$ $|3 - x|$

distance between a and b

If a and b are any real numbers, then the distance between a and b is the absolute value of the difference, namely,

$$|a - b| = |b - a|$$



Example 3

The distance between -4 and -6 is

$$|-4 - (-6)| = |-4 + 6| = |2| = 2$$

Properties of Absolute Values خواص القيمة المطلقة

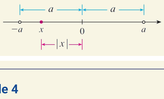
5 Properties of Absolute Values Suppose a and b are any real numbers and n is an integer. Then

1. $|ab| = |a||b|$
2. $\left|\frac{a}{b}\right| = \frac{|a|}{|b|} \quad (b \neq 0)$
3. $|a^n| = |a|^n$

For solving equations or inequalities involving absolute values, it's often very helpful to use the following statements.

6 Suppose $a > 0$. Then

4. $|x| = a$ if and only if $x = \pm a$
5. $|x| < a$ if and only if $-a < x < a$
6. $|x| > a$ if and only if $x > a$ or $x < -a$



Example 4

Solve the following equations and inequilities.

$|3x + 5| = 1$ $\qquad$ $|3x + 2| \geq 4$

$|x - 6| < 2$

Summary

Sets

Intervals

Inequalities

Absolute Value